SUM EDGE-COLORINGS OF SOME COMPLETE TRIPARTITE AND SOME REGULAR GRAPHS

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A proper edge-coloring of a graph is called a sum edge-coloring if it minimizes the total sum of colors on all the edges of the graph ([1]). The aforementioned minimal sum is called the edge-chromatic sum of the graph G and is denoted by $\Sigma'(G)$, and the minimal number of colors needed for a sum edge-coloring is called the edge-strength of the graph G and is denoted by $s'(G)$. In this work, we have obtained the exact values of the edge-chromatic sums and the edge-strength of some complete tripartite graphs. We have also given the values of both parameters for some generalization of cycles $C_n(m)$ defined by Parker in [2].

Main Results. We have proven the following theorems.

Theorem 1. *For any $n \in \mathbb{N}$, we have*

$$\sum{}'(K_{n,n,n}) = \begin{cases} \frac{3n^2(2n+1)}{2}, \text{ if } n \text{ is even},\\ \frac{n(2n+1)(3n+1)}{2}, \text{ if } n \text{ is odd}, \end{cases}$$

and

$$s'(K_{n,n,n}) = \begin{cases} 2n, \text{ if } n \text{ is even},\\ 2n+1, \text{ if } n \text{ is odd}. \end{cases}$$

Theorem 2. *For any $n, m, k \in \mathbb{N}$ such that $n \geq m + k$ and $m \geq k$, we have $s'(K_{n,m,k}) = n + m$ and*

$$\sum{}'(K_{n,m,k}) = \frac{(n+m+1)(km+kn+mn) - m^2 n}{2}.$$

Theorem 3. *For any $n \geq 3, m \geq 1$, if nm is an even number, then $s'(C_n(m)) = 2m$ and*

$$\sum{}'(C_n(m)) = nm^3 + \frac{nm^2}{2},$$

otherwise $s'(C_n(m)) = 2m + 1$ and

$$\sum{}'(C_n(m)) = \frac{m(nm+1)(2m+1)}{2}.$$

References.

1. *Bar-Noy A., Bellare M., Halldórsson M.M., Shachnai H., and Tamir T.* On chromatic sums and distributed resource allocation // Information and Computation. Vol. 140, No. 2, 1998. Pp. 183-202.
2. *Parker E.T.* Edge coloring numbers of some regular graphs // Proceedings of the American Mathematical Society. Vol. 37, No. 2, 1973. Pp. 423-424.