ON THE HIERARCHICAL STRATIFICATION OF ONE FAMILY OF DYNAMICAL SYSTEMS

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The fundamental role in the mathematical modeling of both natural phenomena – biological, ecological, economic, astro- and geophysical, social processes, and technological factors belongs to differential equations and dynamical systems, which provide a mathematical foundation and tools for such investigations. When studying the properties of a dynamical system, its phase space is split into separate trajectories, for which their limiting behavior is then analyzed. Dynamical systems with polynomial right-hand sides of the differential equations forming them deserve special research for the purposes of mathematical modeling.

In the proposed work, which serves as a continuation of the authors' original research, a wide category of systems of differential equations is studied, the right-hand sides of which include polynomial forms of the third and second degrees. Polynomials are mutually reciprocal, which implies that they do not have common roots. Dynamical systems are considered on the extended real plane of their phase variables. The process of studying dynamical systems is based both on classical methods of qualitative, including local qualitative, theory of ordinary differential equations, and on techniques specially developed in the course of this research. The global category of dynamical systems is being stratified into a subfamily structure of a number of levels, or layers, of the hierarchy. For naturally occurring subfamilies, their phase portrait is established, reflecting a complete qualitative picture of their phase trajectories in the enclosed Poincare disk [1, 2]. A catalog of layers of the hierarchy of subfamilies of global category systems has been compiled. The number of consecutive levels varies from three to four for different stratification branches. The upper hierarchical layer contains 10 subfamilies of dynamic systems. For subsequent levels, the bundle is determined by the characteristics of separatrices of singular points and other attributes of phase trajectories. Based on the singular point index, it is shown that dynamical systems belong to the entire global category do not have limit cycles.

The results of the proposed work, along with the additions developed in the course of it to the classical LQTDE methods, are promising both in the context of theoretical and applied research related to mathematical modeling based on dynamic systems.

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