STATIONARY ENERGY EXCHANGE BETWEEN TWO WEAKLY COUPLED VAN-DER-POL OSCILLATORS

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In this work we study stationary energy exchange between two weakly coupled Van-der-Pol oscillators with elastic nonlinearity. The coupling is provided by a linear damping oscillator with anchor string; it's partial frequency being close to that of Van-der-Pol oscillator.

$$\frac{d^2u_1}{dt^2} + u_1 + 8\alpha\varepsilon u_1^{\,3} + 2\beta\varepsilon(u_1 - u_3) + 2\varepsilon(4bu_1^{\,2} - \gamma)\frac{du_1}{dt} = 0$$
$$\frac{d^2u_2}{dt^2} + u_2 + 8\alpha\varepsilon u_2^{\,3} + 2\beta\varepsilon(u_2 - u_3) + 2\varepsilon(4bu_2^{\,2} - \gamma)\frac{du_2}{dt} = 0$$
$$\frac{d^2u_3}{dt^2} + (1 - 2\varepsilon\Delta)u_3 + 2\beta\varepsilon(2u_3 - u_2 - u_1) + 2\varepsilon\eta\frac{du3}{dt} = 0, \qquad (1)$$

The small parameter ε (ε <<1) reflects a weak coupling between the oscillators as well as a weak nonlinearity and damping. Further we suppose that α= b=1, β=1.2, Δ=1.Due to the symmetry of the system there are two nonlinear normal modes: in-phase (*u2=u1*) and out-of-phase(*u1=-u2*). We suppose that initial conditions correspond to excitation of one of Van-der Pol oscillators only that is far from the conditions providing an excitation of both nonlinear normal modes. In the considered case we deal with the solutions which are close to limiting phase trajectories[1] presenting an alternative to normal modes and corresponding to complete energy exchange between different parts of the system.

In a certain range of parameters (for example, γ=1.4 and 0.1<η<0.2) the system under consideration demonstrates the stationary energy exchange under strongly asymmetric initial conditions (all the energy is initially concentrated in one of VdP oscillators). This behavior is similar to that of attractor for rather wide domain of initial conditions. It corresponds to an unusual limit cycle: the phase trajectories with the initial conditions under and over limit cycle are attracted by this state. Such attractor is a limit cycle which is strongly different from those corresponding to nonlinear normal modes mentioned above, well known types of attractors characterized by uniform energy distribution. The letter ones are realized where the parameters turn out to be out of the range (η>0.2 or η<0.1).

Conclusion:

It is shown that the system of weakly coupled Van-der-Pol oscillators demonstrates (in a certain range of parameters) an attractor-like behavior.

References

1. *Manevitch L.I.,Smirnov V.V.* Resonant energy exchange in nonlinear oscillatory chains and Limiting Phase Trajectories: from small to large systems// *Phys. Rev. E* **82**, 2010. Pp. 036602-1 - 036602-9