DIALECTICS IN NONAUTONOMOUS MATRIX POPULATION MODELS: ACCURACY OF CALIBRATION VERSUS CERTAIN PREDICTION

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A great advantage of the matrix model for a discrete-structured population, $\boldsymbol{x}(t) \in \mathbb{R}^n$, is the possibility to calibrate the "projection" matrix $\boldsymbol{L}(t)$ form the data of only two consecutive counts (at time monts t and $t + 1$) and to calculate $\lambda_1(\boldsymbol{L}(t))$, the adaptation measure of he local population under study [1]. This is the power of matrix models as a tool for comparative demography, but here also a methodological problem arise when we have a time series of data and it is necessary to summarise the outcome of the entire observation period. The nonautonomous matrix model represents a finie set of *one-step* matrices, $\boldsymbol{L}(t)$, each yielding its own set of quantitative characteristics of the population, which may even be contradictory in the forecast of its fate.

The contradictions are eliminated by averaging the set M of nonnegative matrices that forms the basic model equation

$$\boldsymbol{x}(t + 1) = \boldsymbol{L}(t)\boldsymbol{x}(t),\ t = 0, 1, \ldots, M - 1, \tag{1}$$

and model logic leads to the problem of *geometric* average [2]. Defined by the life cycle graph for the individuals of a given species, the fixed *pattern* of these matrices deprives this problem of *exact* solution, so that the approximate *pattern-geometric* mean becomes the correct mode of averaging [3] – a novel concept in the theory and practice of modelling biological populations.

For the case where the data bear "reproductive uncertainty" [2] and the calibration yields the whole family of matrices, $\{\boldsymbol{L}(t)\} = \boldsymbol{T}(t) + \{\boldsymbol{F}(t)\}$, at each time step, a heuristic method of averaging is proposed, namely, ***TF**-averaging*. It enables calculating uniquely the pattern-geometric mean of the transition matrices, $\boldsymbol{T}(t)$, hence gaining certain age-specific traits from the stage-structured model and, in particular, answering the question how specifically short a *short-lived* perennial lives.

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References

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