PROBLEMS IN THEORY OF LOW-DIMENSIONAL DYNAMICAL SYSTEMS

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We discuss some problems of the global qualitative analysis of low-dimensional polynomial dynamical systems. To control all of the limit cycle bifurcations in such systems, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cyclesterminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. Such a bifurcation can happen, e. g., in a three-dimensional Lorenz system. But this cannot happen for planar systems. That is why the Wintner–Perko termination principle is applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this method, we have solved, e. g., Smale's Thirteenth Problem for the classical Liénard system. Generalizing the obtained results, we have solved Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the Kukles cubic-linear system and for the general Liénard polynomial system with an arbitrary number of singular points. We discuss also how to apply this approach for studying global limit cycle bifurcations of discrete and continuous Holling-type systems which model the population dynamics in biomedical and ecological systems. Finally, applying a similar approach, we consider various applications of three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.