

WAHBA PROBLEM SOLUTION FOR SATELLITE ATTITUDE DETERMINATION

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Wahba problem [1] is the task of constrained optimization seeking the matrix from SO(3) which maximally converges (basing on the least squares criterion) two sequences of unit vectors. Solution of this task is vital for satellite attitude determination using star trackers.

The majority of classic methods for solution of Wahba problem are based on usage of matrix \mathbf{B} , which is formed based on vectors \mathbf{v} and \mathbf{w} :

$$\mathbf{B} = \sum_{i=1}^n k_i \mathbf{w}_i \mathbf{v}_i^T \quad (1)$$

One of the first analytical solutions of Wahba problem has been proposed by Farrell et. al. [2], who have shown that \mathbf{R}_{opt} coincides with orthogonal matrix \mathbf{W} of polar decomposition $\mathbf{B} = \mathbf{W}\mathbf{S}$, where \mathbf{S} is a symmetrical matrix.

To avoid straightforward calculation of eigenvectors of matrix \mathbf{K} , Schuster et. al. [3] have proposed QUEST (QUaternion ESTimator) algorithm which gives an explicit formula for finding the eigenvector corresponding to the maximal eigenvalue λ_{max} , provided that this value is known. The maximal eigenvalue is searched for as the root of a non-linear algebraic equation which the authors propose to solve using Newton-Raphson method. However, such method of finding eigenvalues is the least stable method, therefore, direct search of eigenvectors of matrix \mathbf{K} gives more accurate results.

References.

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